BC Calculus – Hahn / Limburg
Chapter 9 Quiz 2024-25

Andersen Tannirwedi doesn't even need a calculator to do these problems

Period: 4

	1-2	3-4	5	6	7-8	Totals (not an average)
Implementing Mathematical Practices	4	4		4		4
Connecting Representations			4	4		4
Justification					4	4
Communication and Notation		4			4	4

*This is the **non-calculator section** of the quiz. You must turn this page in before taking out your calculator.*

1. [IMP] A curve is defined by the parametric equations $x(t) = t^2 + 3$ and $y(t) = sin(t^2)$.

Express $\frac{d^2y}{dx^2}$ in terms of t.

$\frac{dy}{dt} = 2t\cos(t^2)$ $\quad \frac{dx}{dt} = 2t$ $\quad \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{\cancel{2t}\cos(t^2)}{\cancel{2t}}$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = \frac{dt}{dx} \cdot \frac{d}{dt}\left(\frac{dy}{dx}\right), \left(\frac{1}{2t}\right)\frac{d}{dt}(\cos(t^2))$

$\frac{1}{2t} \cdot -2t\sin(t^2) = \boxed{-\sin(t^2)} = \frac{d^2y}{dx^2}$

2. [IMP] The position of a particle moving in the xy –plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

$\frac{dx}{dt} = 3t^2 - 6t$ $\quad \frac{dy}{dt} = 6t^2 - 6t - 12$ $\quad$ rest when $\frac{dx}{dt}$ and $\frac{dy}{dt} = 0$

$3t^2 - 6t = 0$ $\quad 6(t^2 - t - 2)$

$3t(t-2) = 0$ $\quad 6(t+1)(t-2)$

$t = 0, t = 2$ $\quad t = -1, t = 2$

$\boxed{t = 2}$

3. [IMP, CN] Write an integral expression that finds the length of the path described by the parametric equations $x = \sin t^3$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$. Do not try to evaluate the integral.

$$\frac{\partial x}{\partial t} = 3t^2 \cos(t^3) \qquad \frac{dy}{\partial t} = 5e^{5t}$$

$$\boxed{\int_0^{\pi} \sqrt{[3t^2\cos(t^3)]^2 + [5e^{5t}]^2}\, \partial t}$$

4. [IMP, CN] Write an integral expression that results in the arc length of the function $y = x^4$ from $x = 1$ to $x = 5$. Do not try to evaluate the integral.

$$\frac{\partial y}{\partial x} = 4x^3$$

$$\boxed{\int_1^5 \sqrt{1 + (4x^3)^2}\, dx}$$

5. [CR] What is the slope of the tangent line to the polar curve $r = 2\theta$ at the point $\theta = \frac{\pi}{2}$?

$r = 2\theta$

$x = 2\theta\cos\theta \qquad y = 2\theta\sin\theta$

$$\frac{\partial x}{\partial \theta} = 2\cos\theta - 2\theta\sin\theta \qquad \frac{dy}{\partial\theta} = 2\sin\theta + 2\theta\cos\theta$$

$$\frac{\frac{dy}{\partial\theta}}{\frac{\partial x}{\partial\theta}} = \frac{dy}{\partial x} = \frac{2\sin\theta + 2\theta\cos\theta}{2\cos\theta - 2\theta\sin\theta} \qquad \frac{dy}{\partial x}\Big|_{\theta=\pi/2} = \frac{2(1)}{-2(\pi/2)(1)} = -\frac{1}{\frac{\pi}{2}} = \boxed{-\frac{2}{\pi}}$$

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Definitely still putting forth my best effort: Anderson Tamiberti
Period: 4

You may use a calculator for this section, ***only after*** *you turn in the non-calculator section.*

For all problems on this section, in order to receive full credit, you ***must*** *write out your math work (with proper notation), in addition to providing the final calculator answer.*

6. [IMP, CR] For $0 \leq t \leq 5$, a particle is moving along a curve so that its position at time t is $r(t) = (x(t), y(t))$, and $r(1) = (2, -7)$. It is known that $\frac{dx}{dt} = sin\left(\frac{t}{t+3}\right)$ and $\frac{dy}{dt} = e^{\cos t}$.

a) Write an equation for the line tangent to the curve at the point (2, -7). at $t=1$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{e^{\cos t}}{\sin\left(\frac{t}{t+3}\right)} \qquad \left.\frac{dy}{dx}\right|_{t=1} = 6.938$$

$$\boxed{y+7 = 6.938(x-2)}$$

b) Find the y-coordinate of the position of the particle at time t = 3.

$$\int_1^3 e^{\cos t}\,dt - 7 = \boxed{-5.418}$$

c) Find the total distance travelled by the particle from time t = 1 to time t = 3.

$$\int_1^3 \sqrt{\left(\sin\left(\frac{t}{t+3}\right)\right)^2 + \left(e^{\cos t}\right)^2}\,dt \qquad \boxed{1.813}$$

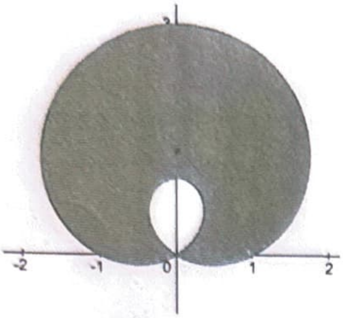
d) Find the time at which the speed of the particle is 2.

$$\text{Speed} = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}$$

$$\sqrt{\left(\sin\left(\frac{t}{t+3}\right)\right)^2 + \left(e^{\cos t}\right)^2} = 2, \qquad \boxed{t = 0.813}$$

*Reminder: For these problems, you **must** write out the full integral expression in addition to providing the final calculator answer.*

7. [CN, J] Find the area enclosed of the shaded region of the graph of $r = 1 + 2\sin\theta$.



$$\frac{1}{2}\int_0^{2\pi} (1+2\sin\theta)^2\, d\theta - 2\cdot\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\, d\theta$$

$$= \boxed{8.338}$$

$1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2} \quad \theta = 7\pi/6,\ 11\pi/6$

8. [CN, J] The figure on the right shows the polar curves $r = 6(\sin(2\theta))^2$ and r = 1 for $0 \le \theta \le \frac{\pi}{2}$.

a) Find the area of the shaded region.

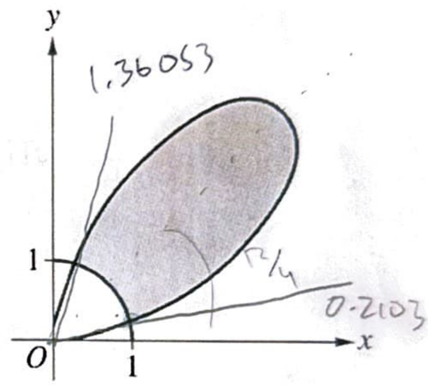
$6[\sin(2\theta)]^2 = 1$

$\sin(2\theta) = \sqrt{\frac{1}{6}}$

$\theta = \dfrac{\sin^{-1}\left(\sqrt{\frac{1}{6}}\right)}{2} = 0.2103$

$$\frac{1}{2}\int_{0.2103}^{1.36053} [6(\sin(2\theta))^2]^2 - 1\, d\theta$$

$$\boxed{= 9.984}$$



$2(\pi/4 - 0.2103) + 0.2103$

$\pi/2 - 0.2103 = 1.36053$

b) Find the perimeter of the shaded region.

$$\int_{0.2103}^{1.36053} \sqrt{[6(\sin(2\theta))^2]^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta + \int_{0.2103}^{1.36053} \sqrt{1+0}\, d\theta$$

$$\left[\frac{d}{d\theta}\left(6(\sin(2\theta))^2\right)\Big|_{x=x}\right]^2$$

$$= \boxed{12.743}$$