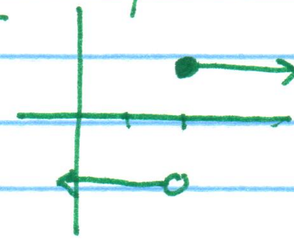
Review Solutions

A 1. I $\dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$ continuous $2+2=2\cdot 2$

II not continuous $\dfrac{x^2-4}{x-2}$ not defined

III Graph



E 2. $3\left(\tfrac{1}{2}\cdot 0 + 2\cdot 9 + 2\cdot 36 + \tfrac{1}{2}\cdot 162\right)$

$\Delta x\left(\tfrac{1}{2}\cdot f(0) + f(3) + f(6) + \tfrac{1}{2} f(9)\right)$

D 3. $\frac{d}{dx}(x+1)^4 = 4(x+1)^3$ this is the definition of $f'(x) = \lim_{h\to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right)$

C 4 not on test!

D 5. $g''(10)=4$ | $g'(x) = 4x+2$
$g'(x) = 4x+C$ | $g(x) = 2x^2+2x+C$
$g'(2) = 8+C = 10$ | $g(0) = 0+0+C = 3$
$C=2$ | $g(x) = 2x^2+2x+3$

C 6. $f'(x) = -3x^2$
$f'(-1) = -3 = m$
$f(-1) = 6-(-1)^3 = 7$

pt-slope: $y - y_0 = m(x-x_0)$
$y-7 = -3(x+1)$
$y = 7-3x-3$
$y = -3x+4$

D 7. at $x=0$: $0^2-1 \overset{?}{=} 0-1$ true

at $x=3$: $3-1 \overset{?}{=} 3^3$ False

B 8. $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ if $f(2)=2^6$ if $f(x)=x^6$

$f'(x) = 6x^5$ $f'(2) = 192$

9. Chain rule!

$y' = 2\sin(2x+3)\cdot\cos(2x+3)\cdot 2$

D Recall $\sin 2\theta = 2\cos\theta\cdot\sin\theta$

$y' = \sin(2(2x+3))\cdot 2 = 2\sin(4x+6)$

10. $W'(x) = \sin x - \cos x$

$W(x) = -\cos x - \sin x + C$ $\qquad \boxed{W(x) = -\cos x - \sin x + 3}$

$W(\pi) = -\cos\pi - \sin\pi + C = 4$

$1 - 0 + C = 4$

$C = 3$

Challenge Problems

1. If $\lim\limits_{x \to 2} \dfrac{x^3 - ax^2 + bx - 2}{(x-2)} = -1$ then $x-2$ is a factor of

$P(x) = x^3 - ax^2 + bx - 2$

$P(x) \div (x-2)$ has $R = 0$

$P(2) = 0$ by remainder thm

$(2)^3 - a(2)^2 + b(2) - 2 = 0$

$\boxed{-4a + 2b + 6 = 0}$

two methods

$$\begin{array}{r|cccc} 2 & 1 & -a & b & -2 \\ & & 2 & 4-2a & 8-4a+2b \\ \hline & 1 & 2-a & 4-2a+b & 6-4a+2b = R = 0 \end{array}$$

$P(x) \div (x-2) = x^2 + (2-a)x + 4 - 2a + b$

at $x = 2$ $\qquad 2^2 + (2-a)\cdot 2 + 4 - 2a + b = -1$

$4a - b = 13$

$-4a + 2b = -6$ $\qquad -4a + 2(7) = -6$

$4a - b = 13$ $\qquad -4a = -20$

$b = 7$ $\qquad a = 5$

2. $f'(x) = 3x^2 - 4x - 5 = -6$ $\qquad$ @ $x = 1/3$ $\qquad$ @ $x = 1$

$3x^2 - 4x + 1 = 0$ $\qquad y - 0 = -6x + 6$

$(3x-1)(x-1) = 0$ $\qquad y - 112/27 = -6(x - 1/3)$

$x = 1/3 \quad x = 1$

3. $f(4) = 1 \quad f'(4) = 1$

a) $f'(4) = 2ax\Big|_{x=4} = 8a = 1/2 \neq 1$

$f(4) = a \cdot 16 = 1 \quad a = 1/16$

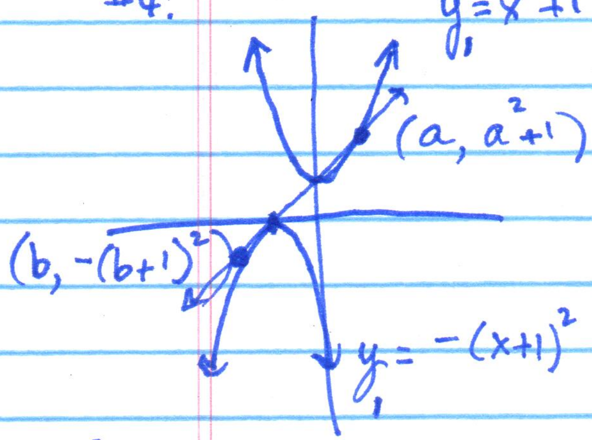
b) $\dfrac{x^n}{k} = 1$ $\qquad h(4) = 1 \quad h'(x) = \dfrac{n x^{n-1}}{k}$

$x^n = k$ $\qquad h'(4) = \dfrac{n \cdot 4^{n-1}}{k}$

$1 = \dfrac{n \cdot 4^{n-1}}{4^n}$

$\boxed{k = 4^4 \qquad 4 = n}$

#4.



You should be able to sketch a graph w/o a graphing calc.

A line tangent to both graphs will have the same slope

$y_1'(x) = 2x \quad m = 2a$

$y_2'(x) = -2(x+1) \quad m = -2(b+1)$

$\therefore\ 2a = -2(b+1)$

$\boxed{a = -b-1}$

$a+b=1$ Eq^{n} 2

use substitution

Slope =

$\dfrac{Y_2 - Y_1}{X_2 - X_1} = \dfrac{a^2+1+(b+1)^2}{a-b}$

$m = 2a = \dfrac{a^2+1+(b+1)^2}{(a-b)}$

$2a^2 - 2ab = a^2+1+(b+1)^2$

$a^2 - 2ab = b^2+2b+2$

$(-(b+1))^2 + 2(b+1)\cdot b = b^2+2b+2$

$\cancel{b^2} + \cancel{2b} + 1 + 2b^2 + 2b = \cancel{b^2} + \cancel{2b} + 2$

$2b^2 + 2b - 1 = 0$

$b = -1/2 \pm \sqrt{3}/2 \qquad a = -(b+1) = -1/2 \mp \sqrt{3}/2$

pt: $(a, a^2+1) \rightarrow -1/2 \mp \frac{\sqrt{3}}{2},\ 2 \pm \sqrt{3}/2$

$m = 2a \longrightarrow -1 \mp \sqrt{3}$

tangent

$y - y_0 = m(x - x_0)$

$y - (2 \pm \sqrt{3}/2) = (-1 \mp \sqrt{3})(x - (-1/2 \mp \frac{\sqrt{3}}{2}))$

$y - 2 \mp \sqrt{3}/2 = (-1 \mp \sqrt{3})(x + 1/2 \pm \sqrt{3}/2)$

$y = (-1 \mp \sqrt{3})x \mp \sqrt{3}/2$

#1. For a proof, use formal definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-h/x(x+h)}{h}$$

$$= \lim_{h\to 0} -\frac{1}{x^2+xh} = -1/x^2$$

2. $x(t) = 100$

$3\sin(\pi t/3) + t^3/2 = 100$, $t = 5.85676$ using solve()

$v(t) = x'(t) = 3 \cdot \pi/3 \cdot \cos(\pi t/3) + 3/2\, t^2$

$a(t) = x''(t) = -3 \cdot (\pi/3)^2 \cdot \sin\left(\frac{\pi t}{3}\right) + 3t$

$a(5.856\ldots) = \boxed{18.062 \text{ ft/s}^2}$

do not round t, use calc to keep all digits

3. $g'(x) = 3x^2/x^4 - 2x/x^4 + \sqrt{x}/x^4 = 3x^{-2} - 2x^{-3} + x^{-7/2}$

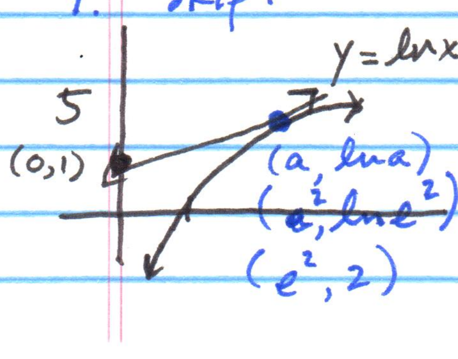
$$g(x) = \frac{3x^{-1}}{-1} - \frac{2x^{-2}}{-2} + \frac{x^{-5/2}}{-5/2} + C$$

$g(x) = -3/x + 1/x^2 - 2/5\, x + C$

$g(1) = -3 + 1 - 2/5 + C = 1$, $C = 3.4$

$$\boxed{g(x) = -\frac{3}{x} + \frac{1}{x^2} - \frac{2}{5}x + 3.4}$$

4. Skip!



$y' = 1/x$ $\quad m = 1/x$ $\quad f'(a) = 1/a$

$$m = \frac{\ln a - 1}{a - 0} = 1/a$$

$\ln a - 1 = 1$

$\ln a = 2$

$a = e^2$

$y - y_0 = m(x - x_0)$

$$\boxed{y - 2 = \frac{1}{e^2}(x - e^2)}$$

$y = \frac{1}{e^2}x + 1$

6. Look for a pattern

$y = \sin 2x$

$y' = 2\cos 2x$

$y'' = -4\sin 2x$

$y''' = -8\cos 2x$

$y^{(4)} = 16\sin 2x$

$y^{4+1} = (2)^5 \cos 2x$

$y^{4+2} = -(2^6)\sin 2x$

$y^{5+2} = -2^7 \sin 2x$

$\vdots$ etc.

$$\frac{d^{97}y}{dx^{97}} = \frac{d^{(4\cdot 24+1)}y}{dx^{(4\cdot 24+1)}} = 2^{97}\cos(2x)$$