Convergence Tests - MC Practice

1. Which of the following series are conditionally convergent?

$i.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ✓

$ii.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ absolutely converges

$iii.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ ✓

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

2. The integral test can be used to determine that which of the following statements about the infinite series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ is true?

(A) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx = -1 + e.$

(B) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx = e.$

(C) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx = 1 - e.$

(D) The series diverges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx$ is not finite.

$-1\int_1^{\infty} \frac{e^{\frac{1}{x}}}{-x^2}\,dx$ $\quad u = \frac{1}{x}$

$du = -\frac{1}{x^2}\,dx$

$-1 \lim_{a \to \infty} \int_1^a e^{1/x}\left(-\frac{1}{x^2}\right)dx$

$-1 \lim_{a \to a} e^{1/x}\Big|_1^a$

$-(e^0 - e^1)$

$e - 1$

3. Consider the series $\sum_{n=1}^{\infty} \overset{a}{\frac{(-1)^n}{\sqrt[3]{n}}}$ and $\sum_{n=1}^{\infty} \overset{b}{\frac{(-1)^n}{n^3}}$. Which of the following statements is true?

(A) Both series converge absolutely. only b

(B) Both series converge conditionally. only a

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges absolutely, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges conditionally.

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges conditionally, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges absolutely.

4. Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. The integral test can be used to verify convergence of the series because $f(x) = \frac{1}{x^2}$ is positive, continuous, and decreasing for $x \geq 1$. Which of the following inequalities is true?

~~(A)~~ $1 + \int_1^{\infty} \frac{1}{x^2}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^2} < \int_2^{\infty} \frac{1}{x^2}\,dx$

(B) $\int_2^{\infty} \frac{1}{x^2}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^2} < 1 + \int_1^{\infty} \frac{1}{x^2}\,dx$

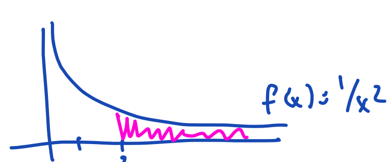
(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} < \int_2^{\infty} \frac{1}{x^2}\,dx < 1 + \int_1^{\infty} \frac{1}{x^2}\,dx$

~~(D)~~ $\int_2^{\infty} \frac{1}{x^2}\,dx < 1 + \int_1^{\infty} \frac{1}{x^2}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^2}$

Bounds

$\int_1^{\infty} \frac{1}{x^2}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^2} < \overset{a_1}{\frac{1}{1}} + \int_1^{\infty} \frac{1}{x^2}\,dx$

if



$\int_2^{\infty} \frac{1}{x^2}\,dx < \int_1^{\infty} \frac{1}{x^2}\,dx$

so $\int_2^{\infty} \frac{1}{x^2}\,dx < \sum_{n=1}^{\infty} \frac{1}{n^2} < 1 + \int_1^{\infty} \frac{1}{x^2}\,dx$

5. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{1}{2^n - n}$ is true?

Ⓐ The series diverges by the nth term test. F

Ⓑ The series diverges by limit comparison to the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$. ← Diverges explore.

Ⓒ The series converges by the nth term test F

Ⓓ The series converges by limit comparison to the geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

$$\lim_{n\to\infty}\left(\frac{1}{2^n - n} \div \frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{n}{2^n - n} = 0$$

L = 0 doesnt go w/ diverge.

$$\lim_{n\to\infty}\left(\frac{1}{2^n - n} \div \frac{1}{2^n}\right)$$

$$= \lim_{n\to\infty} \frac{2^n}{2^n - n} = 1$$

Both converge because L = 1 & $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges

6. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

Ⓐ If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

Ⓑ If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

Ⓒ If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

Ⓓ If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

Ⓔ If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

Direct comparison test

b_n is the mystery

want $b_n < a_n$