Convergence Tests - MC Practice

1. Which of the following series are conditionally convergent?

$i.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$ii.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

$iii.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only

(C) I and III only

(B) I and II only

(D) II and III only

2. The integral test can be used to determine that which of the following statements about the infinite series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$ is true?

(A) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} dx = -1 + e.$

(B) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} dx = e.$

(C) The series converges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} dx = 1 - e.$

(D) The series diverges because $\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} dx$ is not finite.

3. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$. Which of the following statements is true?

(A) Both series converge absolutely.

(B) Both series converge conditionally.

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges absolutely, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges conditionally.

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges conditionally, and $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges absolutely.

4. Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. The integral test can be used to verify convergence of the series because $f(x) = \frac{1}{x^2}$ is positive, continuous, and decreasing for $x \geq 1$. Which of the following inequalities is true?

(A) $1 + \int_1^{\infty} \frac{1}{x^2} dx < \sum_{n=1}^{\infty} \frac{1}{n^2} < \int_2^{\infty} \frac{1}{x^2} dx$

(B) $\int_2^{\infty} \frac{1}{x^2} dx < \sum_{n=1}^{\infty} \frac{1}{n^2} < 1 + \int_1^{\infty} \frac{1}{x^2} dx$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} < \int_2^{\infty} \frac{1}{x^2} dx < 1 + \int_1^{\infty} \frac{1}{x^2} dx$

(D) $\int_2^{\infty} \frac{1}{x^2} dx < 1 + \int_1^{\infty} \frac{1}{x^2} dx < \sum_{n=1}^{\infty} \frac{1}{n^2}$

5. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{1}{2^n - n}$ is true?

Ⓐ The series diverges by the *n*th term test.

Ⓑ The series diverges by limit comparison to the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

Ⓒ The series converges by the *n*th term test

Ⓓ The series converges by limit comparison to the geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

6. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

Ⓐ If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

Ⓑ If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

Ⓒ If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

Ⓓ If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

Ⓔ If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.