3/3 2c. position vector $s(t) = \langle x(t), y(t) \rangle$

FRQ Practice: polar, volume, vector, arc length.

$s(t) = \langle 4\cos(t^2) - 2\sin(t^2)\cos(t^2),$
$4\sin(t^2) - 2\sin^2(t^2) \rangle$

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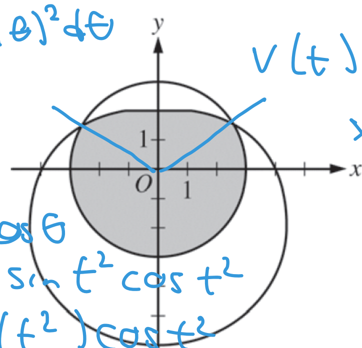
Calculator allowed: 2013

2a) $\frac{2}{3}\pi \cdot 3^2 + \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4-2\sin\theta)^2 \, d\theta$

$= 24.7087$



$v(t) = s'(t) = \langle x'(t), y'(t) \rangle$

$x(t) = 4\cos t^2 - 2\sin t^2 \cos t^2$

$x'(1.5) = -8.072$

$y(t) = 4\sin t^2 - 2\sin^2(t^2)$

$y'(t) = -1.6729$

$v(1.5) = \langle -8.072, -1.673 \rangle$

2b) $x(\theta) = 4\cos\theta - 2\sin\theta\cos\theta$

$x(t) = 4\cos(t^2) - 2\sin t^2 \cos t^2$

$-1 = 4\cos(t^2) - 2\sin(t^2)\cos t^2$

solve by graphing $t = 1.42798$

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2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

polar area $\frac{1}{2}\int_a^b r^2 \, d\theta$

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$x(\theta) = r\cos\theta$, $y(\theta) = r\sin\theta$

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

Extra add ons: arclength Polar $= \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$

(d) Find the perimeter of the shaded region $P = \frac{2}{3} \cdot 2\pi \cdot 3 + \int_{\pi/6}^{5\pi/6} \sqrt{(4-2\sin\theta)^2 + 4\cos^2\theta} \, d\theta$

(e) Find the time when the particle described in part b has a speed of 5.

$P = 17.931$

parametric / vector speed $\sqrt{[x'(t)]^2 + [y'(t)]^2}$

graph directions: define $Y7 = \frac{d}{dx}\left[4\sin(x^2) - 2(\sin(x^2))^2\right]\Big|_{x=X}$

$Y8 = \frac{d}{dx}\left[4\cos(x^2) - 2\sin(x^2)\cos(x^2)\right]\Big|_{x=X}$

do not graph Y7 and Y8

$Y9 = \sqrt{[Y7]^2 + [Y8]^2}$

$Y10 = 5$

Find intersection of Y9 and Y10.

$t = 1.06263, 1.24991$

(b) $A(x)$ of cross sections with radius $r = \frac{f}{2}$

Calculator allowed: 2009 ✓

(a) Area $= 20 \cdot 30 - \int_0^{30} f(x)\,dx$ ✓

$= 218.028 \text{ cm}^2$ ✓

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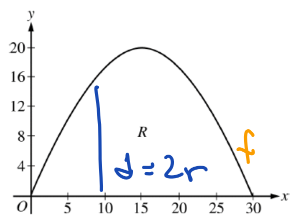

$A(x) = \frac{1}{2}\pi\left(\frac{f}{2}\right)^2$

$V = \int_0^{30} \frac{1}{2}\pi\left(\frac{1}{2}f(x)\right)^2 dx$ ✓ ✓

$= 2356.194 \text{ cm}^3$ ✓

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(c) Perimeter $= 30 +$ arclength

$= 30 + \int_0^{30} \sqrt{1 + (f'(x))^2}\,dx$ ✓ ✓

1. A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.

$= 30 + 51.8037$

$= 81.804 \text{ cm}$ ✓

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(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

(c) Find the perimeter of the base of the cake.

arclength of function $\int_a^b \sqrt{1 + (f'(x))^2}\,dx$

d) $2\pi\int_0^{30} x f(x)\,dx$

$V = 36000 \text{ cm}^3$

Add on for practice:

(d) find the volume of the solid when region R is revolved around the y-axis.

(e) find the volume of the solid when region R is revolved around the horizontal line y=50

washers $\pi\int_0^{30}(50-0)^2\,dx - \pi\int_0^{30}[50 - f(x)]^2\,dx = 101150.444 \text{ cm}^3$

No Calculator allowed: 2011

(a) Perimeter: $1 + k + e^{2k} +$ arclength from 0 to k

$1 + k + e^{2k} + \int_0^k \sqrt{1 + (2e^{2x})^2}\,dx$ ✓ ✓

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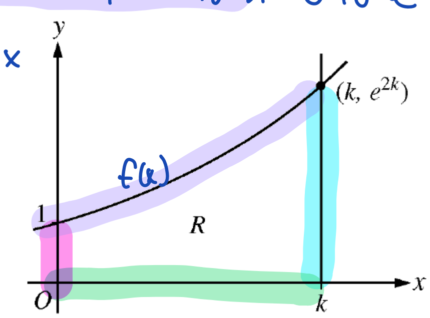

$f'(x) = 2e^{2x}$ ✓

(b) Disks

$V = \pi\int_0^k (e^{2x})^2\,dx$ ✓ ✓

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$= \frac{1}{4}\pi\int_0^k 4e^{4x}\,dx$

$= \frac{\pi}{4} e^{4x}\Big|_0^k = \frac{\pi}{4}\left[e^{4k} - 1\right]$ ✓

3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

(a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

(b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

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(c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

(c) $V(k) = \frac{\pi}{4}e^{4k} - \frac{\pi}{4}$

$\frac{dV}{dt} = \frac{d}{dk}\left[\frac{\pi}{4}e^{4k}\right] \cdot \frac{dk}{dt}$ ✓

$= 4\frac{\pi}{4}e^{4k} \cdot \frac{1}{3} = \frac{\pi}{3}e^{4k}$

$\frac{dV}{dt}\Big|_{k=\frac{1}{2}} = \frac{\pi}{3} \cdot e^{\frac{1}{2}\cdot 4}$

$= \frac{\pi}{3}e^2$ ✓