FRQ Practice 2: Vector, Parametric, Volume, Area, Integration strategies

2011

A graphing calculator is required for these problems.

1. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

   (a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

   (b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

   (c) Find the position of the particle at time $t = 3$.

   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

Calculator allowed: 2008

1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

Add on for practice:

(d) Find the volume of the solid when region R is rotated around the vertical line x = 10

No Calculator is permitted for these problems.

2016

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

   (a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

   (b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

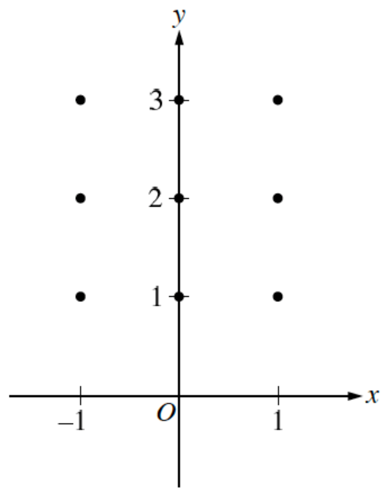
   (c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \to -1}\left(\frac{g(x) - 2}{3(x+1)^2}\right)$. Show the work that leads to your answer.

   (d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

1998

4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



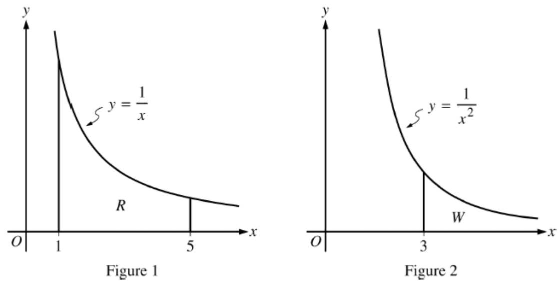
(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

Add on for practice: (d) Find a set of ordered pairs where the particular solution to this differential equation would be constant.

(e) Make a conjecture about the locations of points of inflection for all non-constant solution curves.

2022



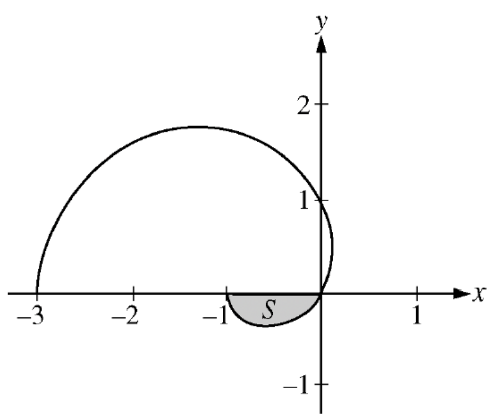
5. Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded by the graph of $y = \frac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \frac{1}{x^2}$ and the x-axis that lies to the right of the vertical line $x = 3$.

   (a) Find the area of region R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.

   (c) Find the volume of the solid generated when the unbounded region W is revolved about the x-axis.

2009



4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

   (a) Write an integral expression for the area of S.

   (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

   (c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.