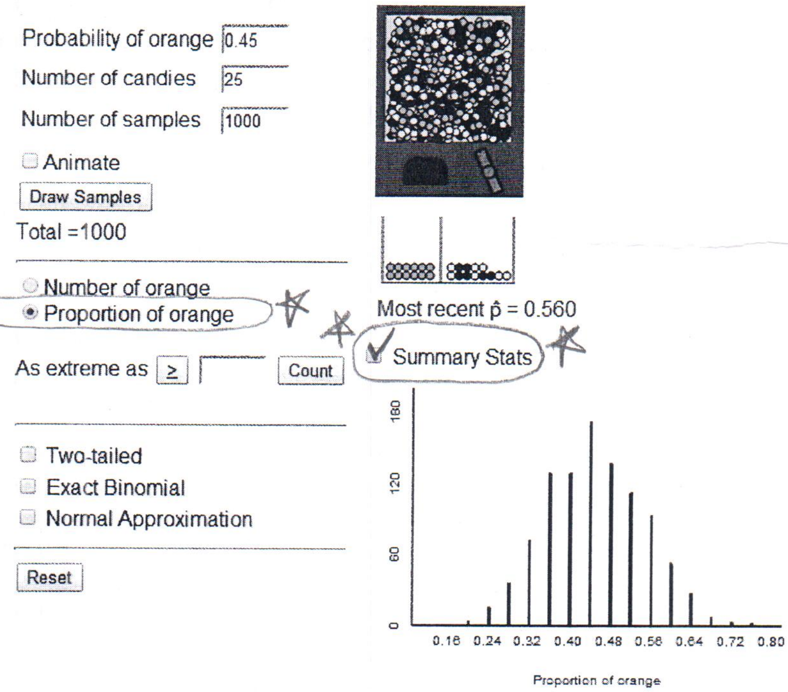
Key

Sampling Distribution of a Sample Proportion

1. Launch the Reece's Pieces applet at www.rossmanchance.com/applets.
   - In the top panel, set the Probability of orange to 0.45, the Number of candies to 25, and the Number of samples to 1. Also, make sure the Animate button is checked and select "Proportion of orange." This will display the sample proportion of orange candies in a sample of 25 from a population with 45% orange candies.
   - Click on Draw Samples a few times and watch what happens. Then, change the Number of samples to 1000, uncheck the Animate button, and press Draw Samples. This applet helps you build an approximate sampling distribution for the sample proportion $\hat{p}$ of orange candies.



We want to learn about the shape, center, and spread of the sampling distribution of $\hat{p}$ for different combinations of n (the sample size) and p (the true proportion of orange candies).

2. Starting with $p = 0.45$ (true proportion of orange = 0.45), let's explore what happens as we increase the sample size n.
   - Check the box for Summary Stats below the candy machine.
   - Set the Number of candies to 5 and the Number of samples to 1000. Make sure the Animate button is unchecked and press Draw Samples. The applet will select 1000 different samples of size 5 and record the sample proportion $\hat{p}$ of orange candiesin each sample. Describe the shape, center (mean), and spread (SD) of the approximate sampling distribution of $\hat{p}$ in the table below.
   - Repeat the previous step for samples of size 10, 25, 100, and 200.

Proportion of orange (p)	Sample size (n)	Shape of approximate sampling distribution of $\hat{p}$	Mean of approximate sampling distribution of $\hat{p}$	SD of approximate sampling distribution of $\hat{p}$
0.45	5	Approx Normal	$\mu_{\hat{p}} = 0.452$	$\sigma_{\hat{p}} = 0.228$
0.45	10	"	$\mu_{\hat{p}} = 0.445$	$\sigma_{\hat{p}} = 0.157$
0.45	25	"	$\mu_{\hat{p}} = 0.446$	$\sigma_{\hat{p}} = 0.099$
0.45	100	"	$\mu_{\hat{p}} = 0.452$	$\sigma_{\hat{p}} = 0.050$
0.45	200	"	$\mu_{\hat{p}} = 0.451$	$\sigma_{\hat{p}} = 0.035$

3. Now, let's keep the sample size the same ($n = 25$) and explore what happens as the value of p changes.
   - Set the Probability of orange to 0.05, the Number of candies to 25, and the Number of Samples to 1000. Make sure the Animate button is unchecked and press Draw Samples. The applet will select 1000 different samples of size 25 and record the sample proportion $\hat{p}$ of orange candies. Describe the shape, center (mean), and spread (SD) of the approximate sampling distribution of $\hat{p}$ in the table below.
   - Repeat the previous step for $p = 0.25$, $p = 0.50$, $p = 0.75$, and $p = 0.95$.

Proportion of orange (p)	Sample size (n)	Shape of approximate sampling distribution of $\hat{p}$	Mean of approximate sampling distribution of $\hat{p}$	SD of approximate sampling distribution of $\hat{p}$
0.05	25	Skewed Right	$\mu_{\hat{p}} = 0.052$	$\sigma_{\hat{p}} = 0.047$
0.25	25	Slightly skewed R	$\mu_{\hat{p}} = 0.249$	$\sigma_{\hat{p}} = 0.085$
0.50	25	Approx. Normal	$\mu_{\hat{p}} = 0.502$	$\sigma_{\hat{p}} = 0.098$
0.75	25	Slightly skewed L	$\mu_{\hat{p}} = 0.749$	$\sigma_{\hat{p}} = 0.088$
0.95	25	Skewed L	$\mu_{\hat{p}} = 0.951$	$\sigma_{\hat{p}} = 0.041$

4. In this part, we will develop a rule for determining when the shape of the sampling distribution of $\hat{p}$ will be approximately Normal.
   - According to your work in Step 2, for what values of n will the sampling distribution of $\hat{p}$ be closest to Normal?

     The larger the n, the more approximately Normal the Sampling Distribution for proportions will become.

   - According to your work in Step 3, for what values of p will the sampling distribution of $\hat{p}$ be closest to Normal?

     The closer p is to 0.5, the more approx. Normal the samp. distr. for proportions will become.

   - Because the shape of the sampling distribution of $\hat{p}$ is based on both n and p, statisticians developed a rule to determine when $\hat{p}$ will have an approximately Normal distribution:

     If $np \geq$ __#___ and $n(1-p) \geq$ __#___, then $\hat{p}$ will be approximately Normal.

   Using the applet, try different combinations of n and p to determine a reasonable value of #.

   $np \geq 10$ $n(1-p) \geq 10$