Significance Testing Practice - One Sample **Name:** Key

AP Statistics

Match the following statements with the words on the right:

1. The null hypothesis is not false, and you rejected it. A	A. Type I error
2. The null hypothesis is not false, and you failed to reject it. C	B. Type II error
3. The null hypothesis is false, and you rejected it. D	C. Correct Decision
4. The null hypothesis is false, and you failed to reject it. B	D. Correct Decision - Power

Explain in your own words:

5. What does the p-value represent?
The p-value represents the probability that a specific outcome happened by chance, assuming that the null hypothesis is true.

6. Why do we have hypotheses in significance tests and not for confidence intervals?
In confidence intervals, we are estimating the value of a population parameter, whereas in a significance test, we are assessing evidence for a claim about a population. We need a hypothesis to test the claim, but not estimate the value.

7. Why do we reject the null when the p-value is small?
If the p-value is small, then it is highly unlikely that the outcome that we are testing could occur by chance under the assumption that the null hypothesis is true. Thus, there is statistically significant evidence that the null hypothesis is false.

8. Why do we "fail to reject the null" instead of "accept the null"?
If we fail to reject the null, we simply do not have enough evidence to reject the null. However, we also do not have enough evidence to accept the null.

9. What would be the probability of making a Type I Error in our problem above? Explain.
The probability of making a Type I Error is α. This is because that is the threshold probability for the null to be rejected, but it might be rejected when it is not false.

10. As part of its 2010 census marketing camping, the U.S. Census Bureau advertised "10 questions, 10 minutes – that's all it takes." On the census form itself, we read, "The U.S. Census Bureau estimates that for the average household, this form will take about 10 minutes to complete, including the time for reviewing the instructions and answers" (with a standard deviation of 4 minutes). We suspect that the actual time it takes to complete the form may be longer than advertised, so we took a random sample of 42 people nationwide (that had completed the form) to ask them how long it took them to fill out the forms. The sample had an average of 12.6 minutes. Is there enough evidence at the 5% significance level to believe that the U.S. Census Bureau overestimated the time it will take to complete the form?

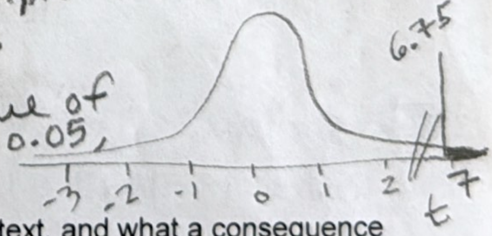
$S_x = 4$ mins

$\alpha = 0.05$

I. $H_0: \mu_f = 10$ $H_a: \mu_f > 10$ μ_f is the true mean time it takes to complete the form (mins)

II. $N \geq 420$ people
SRS - Stated
Normal: $42 \geq 30$ so the data is approximately normal by the CLT

III. Calculations: One-Sample T-Test for a Mean
$df = 41$
$P(t > 4.21) = 6.75 \times 10^{-5}$



IV. Because my p-value of 6.75×10^{-5} is less than $\alpha = 0.05$, I have statistically sig. evidence to reject Ho. Thus, I can conclude that the true mean form time is greater than 10 mins.

a. In the above example, explain what Type I and II Errors would be in context, and what a consequence for each would be.

I: Form doesn't take longer but it is thought to. Marketing campaign not needed.

II: Form takes longer but it is thought that it doesn't. People get frustrated when they expect it to be faster.

11. To test Ho: μ = 105 versus Ha: μ ≠ 105 , a simple random sample of size n = 35 is obtained.

a. Does the population have to be Normally distributed to test this hypothesis? Why or why not?

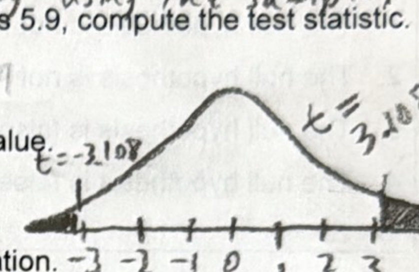
The population does not have to be normally distributed because only the sampling distribution must be approx. normal to test the Ho. This is because we measure the probability using the sample.

b. If the sample mean is 101.9 and the sample standard deviation is 5.9, compute the test statistic.

$t = \frac{\bar{x} - \mu}{\frac{s_x}{\sqrt{n}}}$ $t = \frac{101.9 - 105}{\frac{5.9}{\sqrt{35}}} = \boxed{-3.1084}$ $p = 0.00379$

c. Draw a t-distribution and shade the area that represents the P-value.

$P(-3.11 > t \cup t > 3.11) = \boxed{0.00379}$



d. Find and interpret the P-value in the context of this problem situation.

P=0.00379. Assuming that μ=105, there is a .379% chance that $\bar{x}$=101.9 in a sample of 35, by chance alone.

e. If the researcher decides to test this hypothesis at the alpha = 0.01 level of significance, will the researcher reject the null hypothesis? Why?

Since the p-value (0.00379) is less than α=0.01, the researcher can reject the Ho. There is statistically significant evidence that the population mean is not 105.

12. A cancer research group surveys 500 women more than 40 years of age to test the hypothesis that 28% of women in this age group have regularly scheduled mammograms. Should the hypothesis be rejected at the 5% level if 151 of the women responded affirmatively?

I. α = 0.05

Ho: p = 0.28

Ha: p > 0.28

p: true proportion women > 40 years old who have regularly-scheduled mammograms (said "yes")

II. SRS - assume

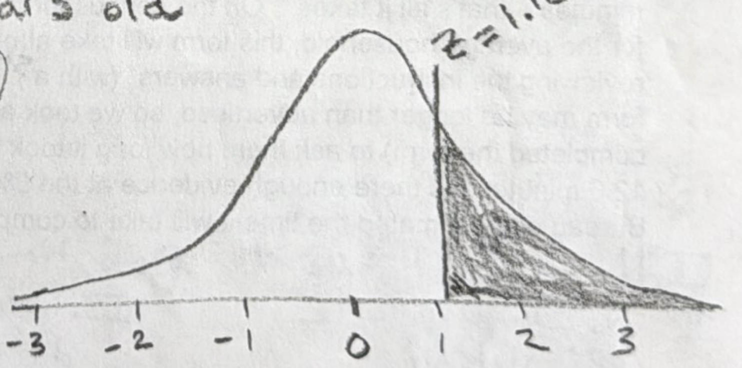
Normal - np = 500 (0.28) = 140 ≥ 10

n(1-p) = 500 (0.72) = 360 ≥ 10   so approx. Normal

N ≥ 5,000 women > 40 years old

$\boxed{\text{1-Prop. Z-Test}}$

III. P(Z > 1.10) = 0.14



IV. Because the p-value of 0.14 is greater than α = 0.05, I do not have statistically significant evidence to reject Ho. Thus, I cannot conclude that the true proportion of women who said "yes" is greater than 0.28.